Smooth Ambiguity Averse Level k

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Abstract

We study the elimination process of dominated bidding prices in a first-price sealed bid auction. In particular, we study the first price auction in a discretized unit interval and construct the upper and lower bounds of feasible bids in the process of elimination of implausible bids, with the help of smooth ambiguity averse model proposed in Klibanoff, Marinacci and Mukerji (2005). We use computational software to compute upper and lower bounds of feasible bids in each round of elimination as well as the convergent stable bids for each type in the discretized unit interval. We compare our result of stable bounds with results from Bayesian Nash Equilibrium for first price auctions.

1 Introduction

Auction theory predicts Bayesian Nash Equilibrium for 1st and 2nd price sealed bid auctions with independent values. For a 1st-price auction with uniform distribution of private values, the unique pure strategy Bayesian Nash equilibrium is half of private valuation when there are two bidders while bidding truthfully is a (weakly) dominant strategy in a 2nd-price auction. But experimental evidence presents results deviating from theoretical equilibria from both directions. For example, Goeree, Holt and Palfrey (2002) found that undergraduate student subjects tended to overbid in first price auctions: they used low and high value treatments for the bidders' valuation and only reported the last 10 periods of experiment sessions to avoid erratic behavior. GHP (2002) found that starting from period 5 subjects' bids would be above theoretical equilibrium (i.e. half of valuation) in both treatments. Underbidding is less common in literature but Cox, Smith, and Walker (1988) found underbidding for low valuations and overbidding in higher valuation in 1st-price auction in one of their experiments [1]. What's more, Garratt, Walker and Wooders (2011) invited highly experienced users in eBay to participate in a 2nd-price auction experiment where values range from 25 to 125 usd uniformly. 38% of the bids were overbids and 41% were underbids. What's more, if subjects had experience of being sellers in the platform, underbid rate would increase to 51% and overbids rate would decrease to 32%.

We might expect experienced bidders to make value bids in the 2nd-price auction and student subjects deviate from theoretical results due to lack of auction experience. And hence it might not be that surprising that undergraduate subjects in GHP(2002) were not bidding according to Bayesnian Nash Equilibrium. But result from GWW(2011) gave the result that over 80% of experienced bidders were deviating from Bayesnian Nash Equilibrium. Such deviation obviously indicates BNE strategies can not do a good job explaining the behaviors of real-life bidders. And hence we will look into auctions via the

[1] Figure 8 in page 84 (a group of 4 bidders)

approach of iterated elimination of implausible strategies: we assume bidders will eliminate prices that are implausible to bid given their beliefs about the auction and publicly available knowledge, in particular, we will assume all bidders know distribution of private values, full rationality of all participants and range of plausible bids. Bidders will accordingly bid prices that survive Iterated Elimination of Implausible prices.

One issue bidders could encounter when using iterated elimination approach is uncertainty over types of opponent and ambiguity on what bidding prices their opponents would be using. A plausible way to address these issues simultaneously is to use the smooth ambiguity averse model proposed in Klibanoff, Marinacci and Mukerji (2005), which enabled bidders to impose subjective beliefs on how opponents' bid would distribute. We will look into a 2-individual first price auction where both bidders believe their opponents would only use pure bids. Smooth ambiguity averse model solved strategic ambiguity by allowing bidders to make subjective beliefs on how opponents' bid would distribute. For simplicity, we may assume that bidders believe their opponents' strategies are distributed uniformly from their current ranges of plausible bids. We justify the use of smooth ambiguity averse model by making comparison with other approaches. The canonical iterated elimination of dominated strategies can not eliminate close-to-zero bids since such a price can be a "best response" to an extreme optimistic scenario where a bidder believes her opponent is also bidding close to zero. So the most significant practical advantage of using ambiguity averse model is the capability of eliminating such extremely small bids: our assumption from the smooth ambiguity averse model where bidders believe the bids her opponents are using distribute uniformly is able to avoid putting all weight on any single (extreme) event. And accordingly we will not have a situation where a bidder believes her opponent is bidding a close-to-zero price with probability 1. What's more, Battigalli and Siniscalchi(2003) constructed upper bound of bids surviving iterated elimination of dominated strategies by assuming bidders are maximizing expected payoff by best responding to belief that opponents are bidding their current upper bound of bids. They proved that the bids that survived iterated elimination will be any prices lower than the BNE result, which is not doing a good job explaining the data.

Another natural approach to address the strategic uncertainty would be maximizing the worst case scenario, i.e. using the maxmin utility function. However, individual with maxmin utility will bid very close to their true valuation, which is also in contrast with empirical data. Iterated elimination of dominated strategies allows bidders to hold various belief on what their opponents may be doing and one particular bids can be justify as long as it is best responding to one belief, regardless of how implausible (impractical) that belief may seem to be. Same issue occurs when we use the maxmin approach, since a belief that opponents are bidding close to private value would also be quite implausible. On the contrary, smooth ambiguity averse model is able to aggregate all possible cases a bidder may encounter evenly through bidders' subjective belief on how her opponents' bids might distribute, if we use a moderate range of ambiguity averse attitudes. KMM(2005) used a concave function $\phi(x) = -\frac{1}{\alpha}e^{-\alpha x}$ where α is the coefficient to measure ambiguity averse attitude, with 0 being ambiguity neutral, positive being ambiguity averse and negative being ambiguity loving. And hence the moderate range of ambiguity averse attitudes indicates a positive but relatively low value for α in function ϕ. KMM (2005)'s smooth ambiguity averse model proved that $E_\mu\phi(E_\pi u \circ b))$ could be used to measure the preferences over act b. According to KMM(2005)'s explanation, π is probability measure on act space and μ measure the bidder's subjective relevance of a particular π as the right probability. In our situation, π is a bidder's belief that her opponent will only bid pure strategies and μ is her subjective belief that her opponent's plausible bids are distributing uniformly over current range of plausible bids.

Recall that bidder will believe that uniform distribution of value types, rationality for all participates in the auction as well as range of feasible bids are public knowledge and will aggregate her ambiguity over opponent's strategies based on such public knowledge. We can now introduce our solution concept: each bidder will construct new upper (lower) bounds by best responding to beliefs that her opponents' pure bids are distributing uniformly from the range of plausible bids when opponents' ambiguity averse attitude reaches the highest (smallest), since the more ambiguity averse a bidder is, the higher bidding price she is likely to use. If θ denotes the public belief that private value follows a uniform distribution when the type space is $[0, 1]$, our utility function should be $E_\theta(E_\mu\phi(E_\pi u \circ b)))$. So the maximizer of $E_\theta(E_\mu\phi(E_\pi u \circ b)))$ when plugging the largest (smallest) ambiguity averse coefficient α into ϕ will be new upper (lower) bounds of plausible bids. Any current plausible price greater (smaller) than the newly computed upper (lower) bound will be called implausible bids and hence will be eliminated. If it is public knowledge that each bidder in the auction knows range of ambiguity averse attitude across all types of bidders are identical, each bidder should be able to compute range of plausible bids in each round recursively. And hence each participant is able to repeat the elimination (optimization) process on the newly computed range of plausible bids until upper and lower bounds converge.

We have constructed our elimination process via dealing with ambiguity averse while we still need to solve some technical issue. Iterated Elimination of Implausible Strategies in a continuum support always has a major drawback: the impossibility to define the smallest increment/decrement. And hence for a 2-individual first price auction with independent values, although people always know that bidder should try to bid only "epsilon" higher than their opponents' highest feasible as upper bound or bid only "epsilon" higher than 0 as lower bound in the current round of elimination of dominated bidding prices, it is practically unfeasible to find such an increment in the bidding space. Battigalli and Siniscalchi (2003) is an example to consider rationalizable bids in first price auction in continuum space. Battigalli and Siniscalchi (2003) assumed that bidders would construct upper bounds of feasible bids by best responding to the case where bidders assumed their opponents would bid their upper bounds from previous round of elimination. But they were only able to eliminate bids from upward and left lower bounds constant at 0. The most straightforward way to avoid the issue encountered in Battigalli and Siniscalchi (2003) is to work on a discrete support. Dekel and Wolinsky (2003) studied an k-individual first price auction with discrete bidding space. Incremental of available discrete bids is $d = \frac{1}{m}$ with m being number of grids in the bidding space. And they succeeded to eliminate all bids except $v_i - d$. Similar to Dekel and Wolinsky (2003), we will simulate our computational process by R-studio. We look at a 2-player first price single-unit auction and discretizing the type space $[0, 1]$ into n grid points with equal grid margin $\frac{1}{n}$. We further assume type and bidding spaces are identical. The available bids for participants in the auction is accordingly the n discrete grid points. We will construct a lower and upper bound for each grid point in the discretized space and compute the bids that survive iterated elimination. Ahn, Choi, Gale and Kariv (2014) discovered that the range of ambiguity averse coefficient is usually $[0, 2]$, with more than 20% of the population being ambiguity neutral and the 95% percentile of ambiguity averse coefficient being only 1.9 or less. So we will be using 2 and 0 as our ambiguity averse coefficients for upper and lower bounds respectively. We also tried to increase the upper bound of ambiguity averse coefficient to larger integers, and the result is higher stable bounds.

The assumption of a uniformly distributed bidding strategy in our model is just for computational simplicity, but also makes our model very similar to the level-k theory. Level-k theory in the context of an auction assumes that in the 1st round, $L0$ participants will bid uniformly and randomly from a range between the highest and lowest prices. While in the next rounds of actions, $L1$ participants will believe that others will behave according to $L0$, and hence they will best respond to such beliefs. A future $Lk(k>1)$ will iterate such type of best response k times, in particular, $L2$ will believe others behaving like $L1$ and best respond to such a belief. We say our model is extending the L1 response to L0 from the level-k theory. The extension we make from Crawford and Iriberri (2007b)'s level-k theory is that bidders in our model construct new ranges of plausible bids by plugging bounds of ambiguity averse coefficients into ϕ function in $E_\theta(E_\mu\phi(E_\pi u\circ b))$. So each of our round is similar to how L1 is best responding to random L0 in level-k theory: bidders in our model construct upper and lower bounds simultaneously with subjective beliefs that opponents' definitive bidding prices distribute uniformly, under the additional condition that bidders themselves have the highest and smallest plausible ambiguity averse coefficients. In a word, our model is eliminating implausible bids and leaving a range of plausible bids every round during the elimination process while after L1 the level-k theory's best response will just be singleton sets. We can name our model "Smooth Ambiguity Averse Level-k" since our elimination process is similar to how L1 responds to L0 in level-k theory and we use smooth ambiguity averse representation to study the elimination process. Crawford and Iriberri (2007b) applied level-k theory into only stage L0, L1 and L2 while ours will solve the whole elimination process.

Section 2 introduces our ambiguity averse version of elimination of dominated prices, and we may call it smooth ambiguity averse level-k to represent that it is a hybrid of the famous smooth ambiguity averse model and level-k theory. We present the numerical results in section and compare it with literature and BNE result in section 3. The last section shows bidding function if we expand the range of ambiguity averse coefficient and compares different solution concepts.

2 Model

We will formally introduce our model in this section. As mentioned, we consider a sealed-bid first price auction with independent private values. We allow 2 ex-ante identical participants in this auction and each player is informed with her private value, v_i, of an indivisible subject. Each bidder submits a price and the object is rewarded to the bidder who bids a higher price. In case of ties, the object will be rewarded equally with 50% probability between the 2 bidders. We further assume that bidding space and value space are equivalent to the discretized unit interval $[0,1]$. The discretized values are from set $V=\{v_1,v_2,...v_{n-1},v_n\}$ and every 2 consecutive grid points v_k,v_{k+1} share the same grid margin $\frac{1}{n}$, i.e. $v_{k+1}-v_k=\frac{1}{n}$ for $\forall k$. So $v_1=\frac{1}{n}$ and $v_n=1$. We assume it is public knowledge that private values have a uniform distribution on the discretized unit interval.

We borrow and extend model from the smooth ambiguity averse model proposed in in Klibanoff, Marinacci and Mukerji (2005), where a double expectation $E_\mu\phi(E_\pi u\circ b)$ is used to measure preferences over acts. In our case, b is the pure bid a bidder is using, and u is the material payoff from the first price single unit auction. ϕ is an increasing transform which characterises altitude towards ambiguity. KMM(2005) derived

$\phi(x) = \frac{1}{\alpha}e^{-\alpha x}$ as a function of constant ambiguity averse with α being the constant absolute ambiguity averse coefficient. The higher this α is, the higher ambiguity averse a bidder will become, which will indicate that a bidder will be more likely to bid a higher price to avoid loss in the auction. We will require the ambiguity averse coefficient $\alpha \in [0, \infty]$. If $\alpha = 0$, we say the bidder is ambiguity neutral while if $\alpha = \infty$ KMM(2005) pointed out a bidder would maxmin preference. We will modify our $\phi(x)$ to be $\phi(x) = \frac{1}{\alpha} - \frac{1}{\alpha}e^{-\alpha x}$ so as to normalize $\phi(0) = 0$. KMM(2005) defined μ to be a subjective probability over the set of probability measures π that a decision maker thought were possible given her subjective information. In terms of our construction of upper and lower bound, π reflects a bidder's belief that her opponent will be bidding a pure strategy and hence should be a degenerate measure on single prices. μ will assign any possible outcome a bidder finds possible a probability. We have assumed that bidders are believing bidders' pure strategy are distributing uniformly, which indicates that μ is assigning probability uniformly to plausible pure bidding prices over range of plausible bids. To be more precise, our model assumes that a bidder with private value v_i will believe her opponent whose private value is one element from the $\{v_1, v_2, ...v_{n-1}, v_n\}$ are bidding pure strategies which are uniformly distributed between the current upper and lower bounds. We extend KMM(2005)'s smooth ambiguity averse model by adding another expectation: π and μ only addresses the scenario when a bidder believes her opponent's valuation is one element from set V, but does not reflect how a certain value is selected from V. The expected utility function will be complete when we introduce another expectation sign outside $E_\mu\phi(E_\pi u \circ b)$. $E_\theta(E_\mu\phi(E_\pi u \circ b))$ with θ being the public belief that private valuations are distributed uniformly on value space. We can restate our model from the most external expectation to the most internal one: a bidder who believes that her opponent's private valuation is uniformly distributed from set V will believe her opponent's bids are pure strategy distributing uniformly from current round of plausible bids.

We should start the elimination process with each bidder (with private value v_i) treating v_1 as the initial lower bound and v_{i-1} as the initial upper bound since bidding the exact private value v_i will not bring positive payoff. The exception here is bidder with private value v_1 who is only able to bid v_1. We call this round 0. We construct upper and lower bounds of feasible in round 1 by solving the following question: a bidder with private v_i will construct the new upper (lower) bound by solving the maximizing question $\max\limits_{b_i \in \{v_1, \ldots, v_{i-1}\}} \sum\limits_{j=1}^{n} \frac{1}{n} \sum\limits_{v_1}^{v_{j-1}} \phi(u(b_i, b_j, v_i, v_j) \frac{1}{j-1}$ when she plugs the highest (lowest) feasible ambiguity averse coefficient into her smooth ambiguity averse representation ϕ. Round 1 will end when bidder with each private value finishes solving these 2 questions. Problem to solve for remaining rounds will be similar to that in round 1 with some change in notations. If we use $lb_m(v_i)$ as lower bound for private value v_i after round m and $ub_m(v_i)$ as upper bound for private value v_i after round m, and denote number of the feasible bids between $lb_m(v_i)$ and $ub_m(v_i)$ as $gap_{m,j}$, (i.e. $gap_{m,j} = n(ub_m(v_j) - lb_m(v_j)) + 1$), the maximization problem in round k will be looking like $\max\limits_{b_i \in \{lb_{k-1}(v_i), \ldots, ub_{k-1}(v_i)\}} \sum\limits_{j=1}^{n} \frac{1}{n} \sum\limits_{b_j = lb_{k-1}(v_j)}^{ub_{k-1}(v_j)} \phi(u(b_i, b_j, v_i, v_j) \frac{1}{gap_{k-1,j}}$. The remaining rounds will continue recursively for every type value until its upper and lower bounds coincide (or each type end up in stable interval of bids).

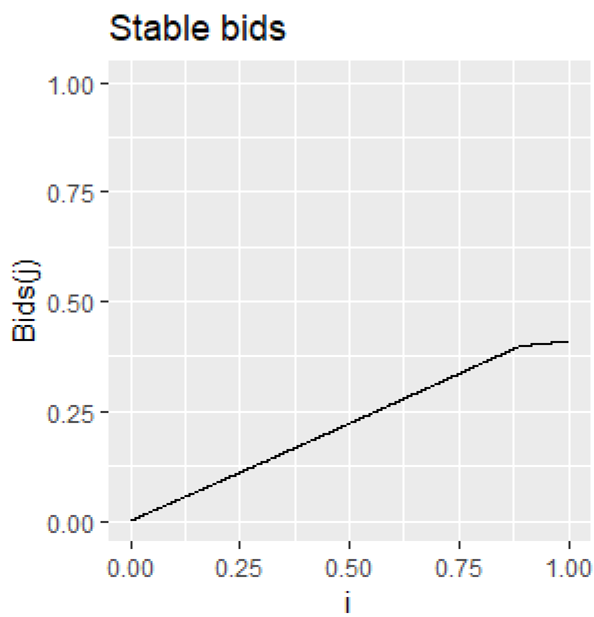
Ahn, Choi, Gale and Kariv (2014) discovered that the range of ambiguity averse coefficient is usually $[0, 2]$, and hence we will compute new upper (lower) bounds to be the maximizers of $E_\theta(E_\mu\phi(E_\pi u \circ b))$ with $\alpha = 2(0)$. When $\alpha = 0$, the $\phi(x)$ will just be x, which will simplify our work to some extent. Our goal is to construct the process of eliminating implausible bids. And we will construct the process with the help of computational software.

3 Results

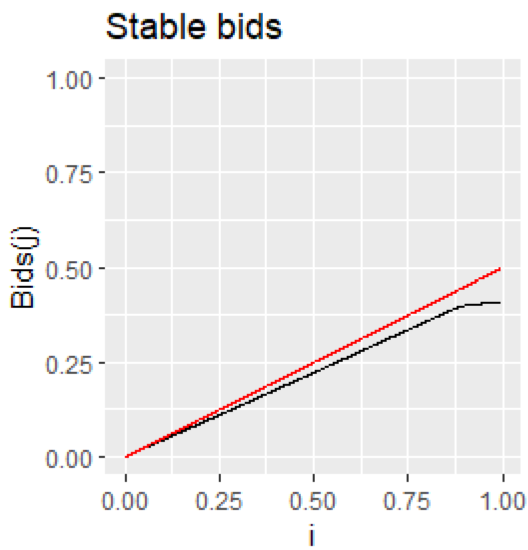
3.1 Stable Bids

Starting from section, we will use terminology "**stable bids**" to describe the bids that upper and lower bounds of plausible bids converge to after rounds of elimination of implausible bids.

The graph of stable bids for $n = 1000$ is

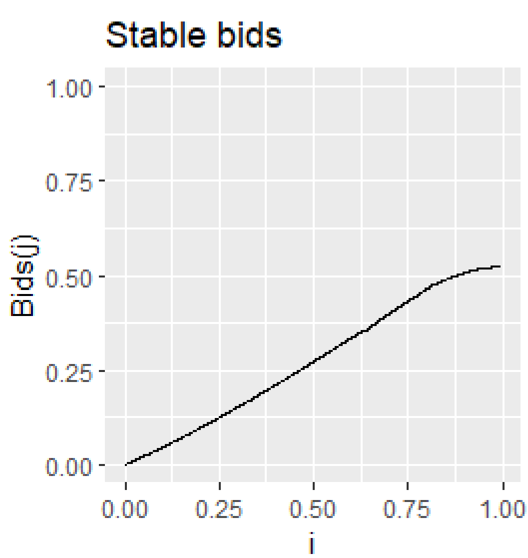


This is how we interpret the graph: the horizontal axis represents value of each type while the vertical axis is the stable bids. A noticeable fact about the bidding function is its significantly concave tail, which indicates bidders with higher private values are underbidding much more than other types. Such fact is reasonable since bidders with higher private values may think themselves have very small probability of loss due to their high private valuations so they choose to decrease their bids to some extent to reflect such confidence as well as to guarantee themselves higher payoff when they win in the auction.



If we compare our results with BNE result (red line), it turns out that we find underbidding for all private values, which means our ambiguity averse reasoning will generate similar result in terms of final bids as GWW(2011). This result should not be surprising since the upper bound of α, 2, is not large. People with small ambiguity averse coefficients would prefer bidding lower with higher net payoff when winning the auction. And they will not weigh a lot on possible situations where they lose the auction by bidding relatively low.

From the graph of stable bids when $\alpha \in [0, 2]$, it seems that we have a close-to-linear bidding function until i reaches 0.800. If we show the graph of stable bids when $\alpha \in [0, 6]$, we have:



It is clear from the graph when upper bound of α gets larger, the bidding function will very close to be linear at first and become concave eventually. And hence our conjecture regarding shape of bidding function is that the bidding function when $\alpha \in [0, 2]$ should have the same shape but the magnitude is relatively small. Such a guess can be checked by fitting the bidding function with polynomials and check the concavity and convexity of the fitted function. And we will check the fitness in the next subsection.

Finally we compare our model to the level-k theory. Our Round 1 is similar to Crawford and Iriberri (2007b)'s random L0, where the latter assumes bidders just bid uniformly from the range of feasible bids. But starting from Round 2 our model drifts away from Crawford and Iriberri (2007b)'s setting where we continue to assume that bidders only know the distribution of plausible bids and they are only able to construct another range of feasible bids by assuming their opponents' ambiguity averse coefficients are the highest or lowest since the more ambiguity averse a bidder is, the more likely she is bidding higher prices. On the contrary, Crawford and Iriberri (2007b) assumed that bidders would choose a strategy which is best response to random L0. Our ambiguity averse elimination process ends after round 6, which is surprisingly short but still longer than L2 (2 rounds) considered in Crawford and Iriberri (2007b). Real participants of auction experiments may not have a deep understanding of the auction and hence they are not able to think their strategies beyond L2. Our model shows that the auctions with bidders who fully understand the mechanism of the auctions will endure longer rounds.

3.2 Fitness

We can run several regressions up to the third power of i to fit the graph for polynomials. The reason we only look at i up to the power of 3 is due to the small absolute value of estimation: the coefficient for i^3 is already 10^{-8} and will only get smaller when we raise the power.

Estimate	Scenario 1	Scenario 2	Scenario 3	Scenario 4
i	0.4372475 ***	4.811e-01***	4.679e-01***	3.585e-01***
i^2		-4.381e-05 ***		2.623e-04 ***
i^3			-3.402e-08***	-2.039e-07***
Constant	2.7686066	-4.554e+00	-4.059e+00	5.700e+00
Adjusted R-squared	0.9976	0.9982	0.9985	0.9992

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

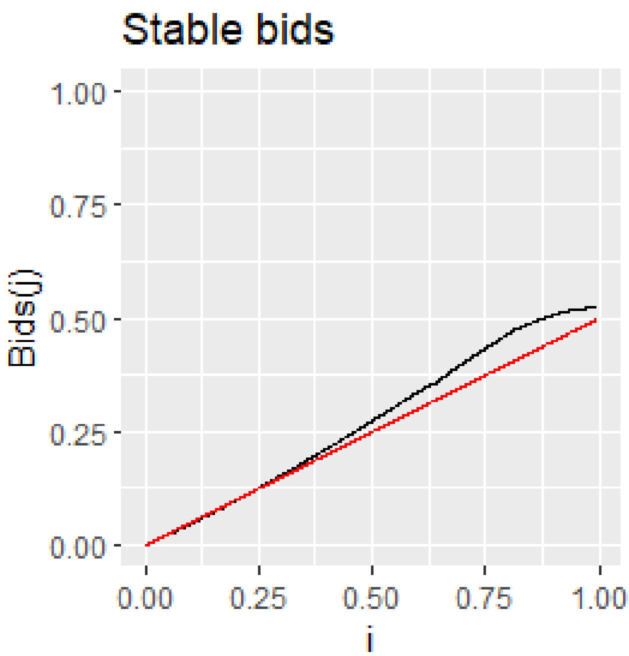
Despite the small absolute values in estimation, statistical significance makes us confident to assert the stable bidding function is not a linear function. And we can fit the bidding function by a polynomial which is slightly convex initially but becomes convex eventually as shown in scenario 3 and 4. The fitness practice seems to justify our conjecture in the last subsection regarding convexity and concavity of the bidding function. If we approximate the bidding function by a polynomial with coefficients computed from scenario 4, firstly it is straightforward to see that when private value is small the bidding function is close-to-linear (actually it is slightly convex). Secondly, we are able to conclude that the polynomial will have a negative second order derivative when private valuation is greater than 0.428 and the second order derivative becomes significantly larger as private valuations increase. This discovery reflects the significantly concave tail we observe in plots of bidding function.

What's more, if we only run regression to approximate bidding functions for types whose private valuations are smaller than 0.5, we have similar estimates for the constant and parameters for i as in scenario 1. And this result confirms our conjecture that the bidding function is close-to-linear when private valuation is small.

4 Discussion

4.1 Selection of Ambiguity Averse Coefficient

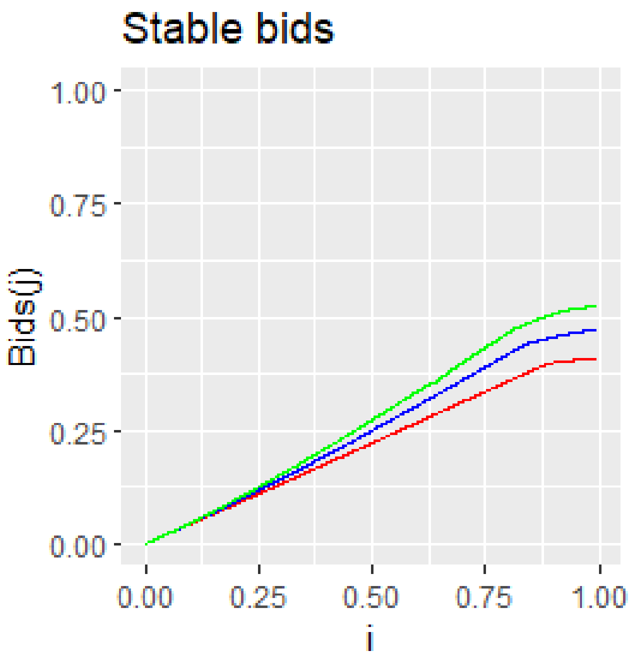
We have illustrated stable bidding functions when upepr bound of α is 2. We can now turn to comment cases with higher α's. If we return to graph when $\alpha \in [0, 6]$:



where the black line is the stable bids from our model and red line is what the Bayesian Nash equilibrium predicts.

We find underbidding for extremely low private values ($i < 0.24$) and overbidding for all other values. This result satisfies our intuitive understanding of the ambiguity averse coefficient: with large ambiguity averse coefficients, people tend to fear that they may lose the auction when they could have won by bidding a higher price. The underbidding case for extremely small private values are symmetric to the "significantly concave tail" phenomenon when α is small. Now bidders with extremely small private values will find themselves next to impossible of winning any auction and they would rather win with some higher payoff if their winning somehow happened and they believe themselves unable to win even if they increase their bids due to small private valuations. Such a finding is consistent with the majority of literature like GHP(2002). The "significantly concave tail" still exists when $\alpha \in [0, 6]$ but shrinks a lot in magnitude, which reflects that when bidders are more ambiguity averse they will not risk losing the auction by bidding relatively lower unless their private values are extremely high. What's more, our $\alpha \in [0, 6]$ setting replicates figure 8 from CSW(1988)'s result where they also found underbidding when private values are small and overbidding when private values are large, except that figure 8 from CSW(1988) was an experiment with 4 participants.

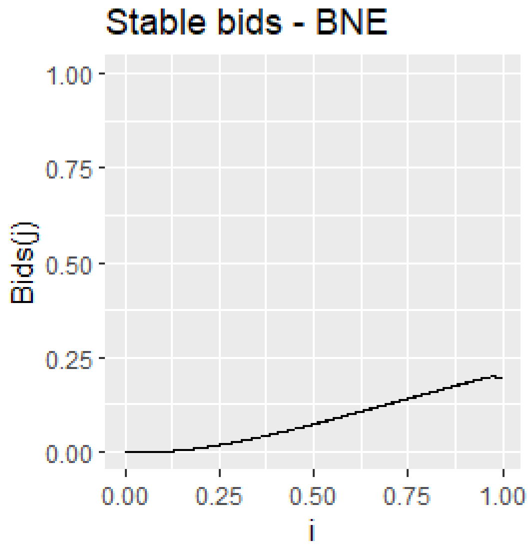
Furthermore, we can report bidding functions with upper bound of α being $2, 4$ and 6 where the green, blue and red lines are bidding function when $\alpha \in [0, 6], [0, 4], [0, 2]$ respectively.



Bidding function will increase for the same value type i as the highest feasible α increases. And hence bidding function goes from underbidding to overbidding as upper bound of α increases. The shape of bidding functions (firstly linear but eventually concave) can be witnessed clearly from the graph. Another observation is that the pivotal point for bidding function turning into concavity from convexity decreases as upper bound of α increases.

Kirchkamp and Reiß (2004) and (2019) studied bidders' behavior in a 2-bidder first-price auction via experiments and one of the auctions had private values distribute uniformly between $[0, 50]$ and only permitted non-negative bids, which was called as "0+" treatment. The "0+" treatment is the traditional first-price auction, which is also consistent with our setting except for the continuum support. "0+" experiment [2] found overbidding for all types of private values and the highest type would overbid the BNE result by 10, which is about 20% of the total range of private values. We can roughly approximate such a result using our model by restricting the ambiguity averse coefficient α to be $[8, 12]$. We can show the difference between stable bidding prices and BNE result in the graph below:

[2] We look at figure 6 in Kirchkamp and Reiß (2004) and 50% quantile line of 0+ treatment (median amount of overbidding) in Fig. 7 in Kirchkamp and Reiß (2019)



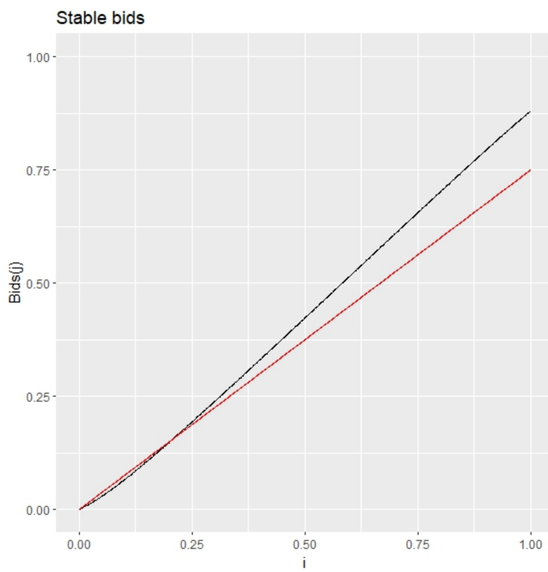
The lower bound $\alpha = 8$ will make sure the low private value types are not underbidding and the upper bound $\alpha = 12$ makes sure the highest overbidding percentage is only 20% of the total range of values.

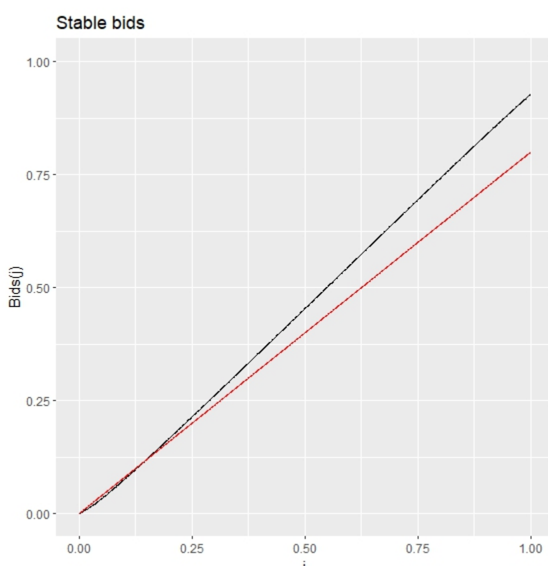
4.2 Comparison with other solution concepts

In this subsection, we want to re-illustrate and emphasize the differences and connections between our model and some existing solution concepts in the literature. The most common solution concept used to study auction is (symmetric) Bayesian Nash Equilibrium. Our approach is obviously not the BNE approach since bids in our model reach stable state (equilibrium) after bidders gradually eliminate bidding prices that are implausible to be best responses, which makes our model very similar to Iterated Elimination of Dominated Strategies. And previous results also dictate strategies that survive Iterated Elimination of Dominated Strategies may not coincide with equilibrium strategies. Our Iterated Elimination of Impossible Strategies reads very similar to Iterated Elimination of Dominated Strategies, but the main difference is that we define upper (lower) bound of plausible strategies as best response to subjective belief that opponents' definitive bids are distributing uniformly when having the highest (smallest) ambiguity averse coefficient while the latter theory eliminates strategies that are never best responses to any belief. Our model and iterated elimination of dominated strategies justify beliefs differently since the latter essentially allow any belief to happen with probability 1 including extreme but rare beliefs, while ours aggregating every possible scenario evenly by imposing an uniform subjective belief on what opponents' bids could be. It would be much easier to eliminate extremely small (and high) bids in our model since to support such bids as best response bidders usually need to come up with a rare event with probability 1.

Maxmin approach is to maximize the smooth ambiguity averse representation under the worst possible scenario and in terms of auction the worst scenario usually means bidders believe that opponents are bidding their highest feasible bids. Interestingly, KMM(2005) pointed out that the maxmin preference is a special case of ambiguity averse model where bidders' ambiguity averse attitudes rise to infinity. Intuitively speaking, the higher the ambiguity averse coefficients, the more likely it is for bidders to focus on cases where where they could have won if they had increased their bids. (Bidders with small or mild ambiguity averse coefficients do not fear of the case above and hence they will bid smaller prices than bidders who are more ambiguity averse.) And hence such bidders will tend to bid close to private values to avoid potential losses when they are extremely ambiguity averse. If we simulate maxmin preference in smooth ambiguity averse model, we can plug very high ambiguity averse coefficients α into the ϕ function and let bidders believe opponents are bidding the upper bounds of plausible bids. We should expect to see extremely high bidding prices. For example, the highest bid will be 88% of private values if we pick $\alpha \in [40, 50]$ and be higher than 90% if we pick $\alpha \in [90, 100]$. Majority (more than 75%) of stable bids will be higher than 75% of private value if $\alpha \in [40, 50]$ and higher than 80% of private value if $\alpha \in [90, 100]$.

We show the two bidding functions in graphs below, The first graph is when $\alpha \in [40, 50]$ and the second is when $\alpha \in [90, 100]$. The black lines represent stable bids predicted by our model while the red lines represent bidding 75% and 80% of private valuations respectively in each plot.





But such close-to-private-value bidding prices are never observed in experiments and neither do participants in real experiments own extremely high ambiguity averse attitudes . Ahn, Choi, Gale and Kariv (2014) discovered that the range of ambiguity averse coefficient is only $[0, 2]$. In conclusion, we do not select maxmin utility since literature finds it very rare for real individuals to have extreme ambiguity averse attitudes.

The last solution concept we want to compare is level-k theory. We mentioned in the introduction section that the main difference between our model and level-k is that we introduce a range of ambiguity averse coefficients so that we are able to construct upper and lower bounds by using the highest and lowest ambiguity averse coefficient. We can accordingly treat Crawford and Iriberri (2007b)'s level-k theory as an extreme case of our smooth ambiguity averse level-k model where the upper and lower bound of ambiguity averse coefficients are set to be identical at 0. If $\alpha = 0$, $\phi(x)$ is easily proved to be identity function, which makes $\phi \circ u$ the material payoff function used in Crawford and Iriberri (2007b). An L0 bidder is defined to bid uniformly from the plausible set of prices and L1 is best responding to L0, which is similar to our first round of elimination of implausible bids where bidders believe opponents' definitive bids are distributing uniformly. But Lk's best response to L(k-1) for any $k \geq 1$ will only be a singleton set of bidding prices since level-k theory is essentially that the upper and lower bound of ambiguity averse coefficient are both 0. According to the description above, we may view level-k theory as a very specific case of our smooth ambiguity averse level-k model. A difference between our model and level-k theory in Crawford and Iriberri (2007b), however, is that they stopped their study at L2, which is only 2nd round since they thought that experiment subjects may not be able to think beyond that level. Ours will not stop until equilibrium is reached.

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